## QUESTION 1

Recently, Taco Bell announced that it would discontinue all of its potato products. This made Karthik, a National Taco Day ambassador, very sad, because Karthik always enjoyed **sum** nice sides of Cheesy Fiesta Potatoes. Please evaluate the following summations.

$$\begin{aligned} A &= \sum_{n=0}^{\infty} \frac{1}{2^n} \\ B &= \sum_{n=1}^{\infty} \frac{1}{3^n} \\ C &= \sum_{n=0}^{\infty} \frac{2n}{3^n} \\ D &= \sum_{n=0}^{\infty} \frac{1}{n^2+5n+6} \end{aligned}$$

Compute $AB + CD$.

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## QUESTION 2

During the Adventures of Mighty Macchar, Yung Haldi, and Dr. Dispute (Mr. Macchar's sidekicks) the three of them are very determined to inverse the wrongdoings of Dr. Doofenshmirtz! While they're at it, they find the following matrix $M$:

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ 4 & -2 & 1 & 6 \\ 2 & 5 & -6 & 2 \\ 3 & 1 & 7 & 8 \end{bmatrix}$$

If $N = M^{-1}$, help the gang out by finding the value of Doofenshmirtz's password: $N_{2,1} + N_{3,3}$.

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## QUESTION 3

Ishrit enjoys politics, and often gets into arguments that he shouldn't get into for that reason. As you know, **pol**iticians enjoy solving **pol**ynomial equations in their free time. Help Ishrit solve the following questions about polynomial equations.

$$A = \text{The remainder of the polynomial division } \frac{x^2+5x+8}{x+2}$$

$$B = \text{The distance from the origin to the intersection point of the horizontal and vertical asymptotes of } \frac{3x^2+6x-9}{x^2+7x+12}$$

$$C = \text{The x-intercept of the oblique asymptote of } \frac{60x^3+261x+104x^2+73}{30x^2+7x+53}$$

$$D = \text{The ordinate of the removable discontinuity of } \frac{x^3+4x^2+x-6}{x^2+3x-4}$$

Calculate $ABCD$.

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Calculate $ABCD$.

## QUESTION 4

Rohan's knees remind many of trees. Speaking of trees, evaluate the following expressions related to **log**arithms and exponents. Hint: $\log 2 = 0.30103$

$$A = \log_{0.01} 0.000001$$
$$\text{B} = \text{The number of digits in } 5^{1000}.$$
$$C = \log 6 + 2\log 5 + \log 4 - \log 3 - \log 2$$
$$D = \frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz}$$

Calculate A + B + C + D.

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## QUESTION 5

Saathvik and Ian find Daniel to be quite eccentric. Help Saathvik and Ian calculate the eccentricity for the following ellipses.

$$\text{A} = \text{Eccentricity of } 9x^2 + 25y^2 - 18x - 100y - 116 = 0$$
$$\text{B} = \text{Eccentricity of } 25x^2 + 16y^2 = 400$$
$$\text{C} = \text{Eccentricity of } x^2 + 4y^2 = 4$$
$$\text{D} = \text{Eccentricity of } 4x^2 + 9y^2 = 36$$

Calculate $ABCD$.

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Calculate $ABCD$.

## QUESTION 6

Ramya's interest in math is expanding! Help her find details about each of the following expansions:

$$A = \text{the coefficient of the third term in the expansion of } (x-3)^8$$
$$B = \text{the constant term in the expansion of } (2x + \frac{1}{x^2})^6$$
$$C = \text{the sum of the coefficients in the expansion of } (x+4y+z)^3$$
$$D = \text{the number of terms in the expansion of } (x+3y+2z)^8.$$

Calculate $A - B + C - D$.

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## QUESTION 7

One day, Akash is listening to his favorite anime music when his Spotify account is hacked by Mihir and Farzan. He is shocked when the following message pops up on his screen.

IN ORDER TO LISTEN TO YOUR MUSIC, YOU HAVE TO SOLVE THE FOLLOWING EQUATION FOR CERTAIN VALUES. GYAHHH.

$$13x^5 + 7x^4 - 10x^3 + 9x^2 + 2x = 10$$

A = The reciprocal of the sum of the roots
B = The sum of the reciprocals of the roots
C = The sum of the roots, taken 3 at a time
D = The sum of the squares of the roots

Calculate $\frac{ABD}{C}$.

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## QUESTION 8

Let:

$$A = \log B$$
$$B = D^{log_8 64}$$
$$\frac{1}{\log_2 C} + \frac{1}{\log_3 C} + \frac{1}{\log_4 C} = 1$$
$$D = C \cdot \log_{4096} 2$$

Calculate A to the nearest hundredth.

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## QUESTION 9

Shreyas is known to have a very **hype** personality and also Chronic Conic Syndrome, where he does not enjoy evaluating properties of conics. Help Shreyas calculate the following properties of the **hype**rbola below.

$$9x^2 - 16y^2 = 144$$

A = Eccentricity of the hyperbola

B = Distance between foci

C = Length of conjugate axis

D = Length of Latus Rectum

Calculate $\frac{BCD}{A}$.

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## QUESTION 10

Nitish and Sruthi are playing tennis when they find out that the trajectory of the balls they hit happens to be that of a parabola! The parabola has a focus at $(0, 0)$ and the equation of its directrix is $x = 5$. If the parabola is expressed in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ for positive $C$ and integer coefficients, find the distance between the points where it intersects the conic $(A + 1)x^2 + 2Bxy + (C + 1)y^2 + 2Dx + 2Ey + 3F = 0$.

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## QUESTION 11

Dylan really loves arithmetic and geometric sequences, and is constructing one on his desk, but the bell rings before he can finish it! Prabhas happens to sit in the same spot during the next period and is given the task of finding the $17^{th}$ and $24^{th}$ elements of the sequence. Unfortunately, all Dylan wrote down was that the $10^{th}$ element is 38, and the $31^{st}$ element is 1026. Let:

$$A = \text{the } 17^{th} \text{ element if the sequence is arithmetic.}$$
$$B = \text{the } 17^{th} \text{ element if the sequence is geometric.}$$
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Calculate $A - B + C - D$.

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## QUESTION 12

Tanusri and Akash catch themselves in a verbal altercation where Akash ends up savagely roasting Tanusri by ending the argument with "Get Real." Infuriated by this comment, Tanusri decides to find out if the four polynomials below have real roots or not by making a binary Key with each bit depending on whether the polynomial has at least one real root. For example, if the first two did have at least one real root, but the last two did not, the Key would be 12 ($1100_2$ in base 10). Find the sum of all integer values of $a$ for which $|a| \leq 30$ and the Key is a positive perfect square.

$$ax^2 + 3x - 9$$
$$9x^2 - ax + 1$$
$$-x^2 + 5x + a$$
$$x^2 + ax + 4$$

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Rayyan believes that the only valid way to solve a system of equations is by using Cramer's Rule. Using Cramer's rule, help him find the sum of the determinants of the x, y, and z matrices in the following system of equations.

$$2x - y - 8z = 11$$
$$4x + 6y + 13z = -18$$
$$10x + 7y + 11z = -5$$

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## QUESTION 14

Most people don't actually know this, but Eric has an excellent singing voice. He is said to have the best **harmony** in all of Tallahassee! Help Eric find the harmonic means of the following numbers.

A = the harmonic mean of 1, 2, 5, and 6.
B = the harmonic mean of 3 and 9.
C = the harmonic mean of 4 and 6.
D = the harmonic mean of 5 and 7.

Find the product $ABCD$.

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Find the product $ABCD$.

## QUESTION 15

Tanmay enjoys eating ice cream, but has very sensitive teeth. Because of his teeth being so sensitive, he has to wait for the ice cream to thaw before he can take it out of its container and eat it. He recently learned that the rate of ice cream melting is equal to the **exposed** surface area of the ice cream over the total volume of the ice cream. Assume that the only exposed surface area is the top, which has dimensions of length by width. Given the length, width, and height respectively of the following ice cream containers, find the order of the thaw rates of the ice cream, from greatest to least. For example, if they are given in decreasing order, your answer should be A, B, C, D. Additionally, assume all units are uniform.

A = 4, 5, 6
B = 9, 3, 8
C = 7, 9, 20
D = 2, 3, 4

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